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Measuring and Optimizing Conditional Value at Risk Using Copula Simulation

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Abstract. Copula models have increasingly become popular for the modeling of the dependence structure of financial risks. This is due to the fact that in contrast to linear correlation, a copula can model the complete linear and non-linear dependence structure of a multivariate distribution. This paper discuses methods for measuring and optimizing the market risk of a portfolio using copula simulation. Particularly, it is aimed to model accurately joint distribution of stock prices using pairwise bivariate copula. In order to achieve this aim, the return of each stock price is characterized individually. The distribution of each return series is fitted semi-parametrically using a piecewise distribution with generalized Pareto tails. To better characterize the behavior in each tail, Extreme Value Theory (EVT) is applied. The lower and upper Pareto tails are estimated parametrically, while the interior is estimated by non-parametric kernel-smoothed interior. By this way a composite semi-parametric CDF for each stock price is constructed. The dependence structure of assets in portfolio is modelled by copula functions. Five types of dependence structure are proposed: the Gaussian copula, t-copula, Frank copula, Gumbel copula, and Clayton copula. The aim of this paper is optimize the conditional value at risk (CVaR) based on ss the impact of each type of copulas. To achieve this goal, four Indonesian Blue Chip stocks: ASSI=Astra International Tbk, BMRI=Bank Mandiri (Persero) Tbk., PTBA=Tambang Batubara Bukit Asam Tbk., INTP=Indocement Tunggal Perkasa Tbk recorded during the period of 6-11-2006 to 2-08-2013 are used to compose the stock portfolio. The VaR of the stock portfolio is minimized. The outcomes of this experiment is in terms of optimal portfolio compositions.
Keywords: VaR, CVaR, copula, optimization, dependence structure, Monte Carlo Simulation.

1 Introduction

According to some research about financial data that the stock prices clearly show the non-normality, such as asymmetry, leptokurtosis or fat tails of their return distributions. Fat tail of the data means that the probability of extreme events which are observed empirically is higher than the probability of extreme events under the normal distribution. One may refer to [16] for the formal definition about fat tailed distribution. Tail dependence refers to the extent to which the dependence among stock prices arises from extreme observations or events.

In order to include these empirical characteristics of stock price distributions, particularly to model the tail dependence among the stock prices, a copula simulation model for generating the portfolio loss distribution is performed. In particular, in this model, the dependence structure for the asset in portfolio is described by different types of copula functions. In this paper, the impact of different types of copula function both on portfolio market risk measurement and on the efficient portfolio composition (or portfolio optimization) is tested, in particular the uses of copula function to simulate market risk.

This study follows the research contributions by Clemente and Romano [8]. In their research, default events for credit risk are modeled using copula functions and the loss distribution of the loan portfolio is generated by Monte Carlo simulation. However, this paper stresses on modeling market risk of a stock portfolio composed by using pairwise bivariate copula. In addition to Clemente and Romano [8], copula functions are applied to capture multiple correlations. This paper is to try to adopt the archimedean and elliptic copula families in modeling the dependence structure of market price of assets and to see how copula perform in the minimization of the portfolio. For these purposes, five types of dependence structures are applied to model the dependence structure. The studies in [5],[13], [14], underline the ability of the copula function to take into account the tail dependence present in a large set of historical risk factor data, in particular where the risk factors are of very different type.

This paper emphasizes the uses of copulae as they are useful tools to be implemented efficiently in simulating the multi-variate distribution. In fact, copula functions can be used to model the dependence structure independently of the marginal distributions. Using copulae, separating dependence and marginal behavior of the asset returns in portfolio are possible. By this method, a multivariate distribution with different margins and a dependence structure can be constructed. Particularly, copula function can join the evolution of asset returns and account for tail dependence. The most important thing is by using copula function, the variation of tail dependence can be modeled in more flexible approach ([7] and [6]). In our context, tail dependence refers to the occurrence of extreme event of asset price in portfolio.

Furthermore, this paper also applies the supplement (or alternative) to VaR, that is the Conditional Value-at-Risk. The CVaR risk measure is closely related to VaR. For continuous distributions, CVaR is defined as the conditional expected loss under the condition that it exceeds VaR, see [18] and [19]. For continuous distributions, this risk measure also is known as Mean Excess Loss, Mean Shortfall, or Tail Value-at- Risk. However, for general distributions, including discrete distributions, CVaR is defined as the weighted average of VaR and losses strictly exceeding VaR, see [18] and [19]. Recently, the redefinition of expected shortfall similarly to CVaR can be found in [1]. For general distributions, CVaR, which is a quite similar to VaR measure of risk has more attractive properties than VaR. CVaR is sub-additive and convex explained ([18] and [19].

Moreover, CVaR is a coherent measure of risk discussed in [2] and proved in [17].

After generating the different portfolio loss distributions derived from the copula functions, the Conditional Value-at-Risk (CVaR) are estimated for 95-percent and 99-percent confidence level and over a time horizon of one day. In this case VaR and CVaR are optimized using computationally efficient methods such as linear programming. In order to minimize portfolio CVaR, *PortfolioCVaR* object of MATLAB R801 is implemented subject to the traditional constraints of balance, objective expected return. The well-known frontier of efficient portfolios is established. Furthermore, it is also demonstrated that the minimization of CVaR also produces a good reduction of VaR.

2 Generalized Pareto Distribution Copula Approach

Copula describes the dependence structure of random variables. Copula binds together the probability distributions of each random variable into their joint probability. Sklar's theorem (see [7]) provides the theoretical foundation for the application of copulas.

Definition 1. *A $d-$dimensional copula function C is a multivariate c.d.f. with margins uniformly distributed on $[0,1]$ and with the following properties:*

(1) $C : [0,1]^d \to [0,1]$;
(2) C *is grounded and* d *is increasing;*
(3) C *has margins* $C_i,\ (i = i, \ldots, d)$ *satisfying:*

$$C_i(u) = C(1, \cdots, 1, \cdots, u, 1, \cdots, 1) = u$$

for all $u \in [0,1]$.

C is the unique copula associated with the distribution function. Let $F(x_1, \cdots, x_d)$ be a joint distribution function with marginal distribution functions $F_1(x_1)$, $\cdots$, $F_d(x_d)$, the copula associated with F is a distribution function $C : [0,1]^d \to [0,1]$ that satisfies

$$F(x_1, x_2, \ldots, x_d) = C(F_1(x_1), \cdots, F_d(X_d); \theta) \tag{1}$$

where θ is the parameter of the copula called the *dependence parameter*, which measures dependence between the marginals. The basic idea behind copulae is to separate dependence and marginal behavior of the univariates. To understand easily, a $2-$dimensional copula function may be considered also known as **bivariate copula**. One may refer to [7] and [15] for detail discussions about bivariate copula.

Definition 2. *The bivariate copula is a function* $C : [0,1]^2 \to [0,1]$ *with the following properties:*

1. *For every* $u_1, u_2 \in [0,1]$ $C(u_1, 0) = 0 = C(0, u_2)$.
2. *For every* $u_1, u_2 \in [0,1]$ $C(u_1, 1) = u_1$ *and* $C(1, u_2) = u_2$.
3. *For every* $(u_1, u_2), (u_1', u_2') \in [0,1]^2$ *such that* $u_1 \leq u_2$ *and* $u_1' \leq u_2'$

$$C(u_2, u_2') - C(u_2, u_1') - C(u_1, u_2') + C(u_1, u_1') \geq 0$$

Empirical evidence suggests that stock returns exhibit higher kurtosis ('fat tails') than the normal distribution (see [4]). Extreme value theory (EVT) can be applied to estimate the tails of abnormally distributed marginal density functions. (see [9], [10], [11]).

Applying EVT as in [9], the probability $\mathbb{P}$ that returns X will exceed high thresholds λ is approximately

$$\mathbb{P}(X - \lambda | X > \lambda) \approx G(y) \tag{2}$$

where $G(y)$ represents a GPD:

$$G_\xi(y) = \begin{cases} 1 - (1 + \xi y)^{-1/\xi} \text{ for } \xi \neq 0, \\ \\ 1 - e^{-y}, \qquad\quad \text{ for } \xi = 0; \end{cases} \tag{3}$$

with $y \geq$, for $\xi \geq 0$, and $0 \leq y \leq -1/\xi$ for $\xi < 0$. GPDs can be applied to model the tail behavior of stock returns exceeding high thresholds. The parameters of GPDs are usually estimated by applying the maximum likelihood method (see [9]).

As all continuous variables can be transformed into uniformly distributed variables, it is sufficient to define copulas $C(u_1, u_2, \cdots, u_d)$ specifying the dependence structure between uniformly distributed variables u_i. The appropriate copula would be the one that is able to capture the dependence structure constructing the margins. The dependence structure embodied by the copula can be recovered from the knowledge of the joint distribution F and its margins F_i. The estimation of all parameters of the copula function are estimated by performing procedures described in [7]. See MATLAB documentation, [12] for more details with regard to the estimation of the copula parameters and the algorithm for simulating Monte Carlo scenarios.

3 Optimization of Value at Risk

Consider a portfolio consist of d stocks and denote by w_i the weight of stock i allocated to the portfolio at time t. Let $f(\mathbf{w}, \mathbf{r})$ be the loss function of the portfolio, with $\mathbf{w} \in \mathbb{R}^d$ is the vector of the portfolio and $\mathbf{r}$ is the vector of asset returns. Let ξ be a certain threshold, the Value-at-Risk of a portfolio at level α is defined as the lower $\alpha-$quantile of the distribution of the portfolio return

$$\text{VaR}(\alpha) = \inf\{\xi \in \mathbb{R} : \mathbb{P}(f(\mathbf{w}, \mathbf{r}) \leq \xi) \geq \alpha\} \tag{4}$$

The uses of VaR to measure financial risks have been increasingly popular since 1990s ([10] and [11]). VaR now becomes the standard risk measure used by financial analysts to quantify the market risk of an asset or a portfolio.

CVaR is a supplement or an alternative to VaR. CVaR is another percentile risk measure which is called Conditional Value-at-Risk. For continuous distributions, CVaR is defined as the conditional expected loss under the condition that it exceeds VaR. The following approach to CVaR is summarized from [18] and [19]. Let r_p be the return of the portfolio, then r_p is defined as a random variable satisfying $r_p = w_1 r_1 + w_2 r_2 + \cdots + w_d r_d = \mathbf{w}^{\mathrm{T}}\mathbf{r}$ and the weight constrain condition is imposed to be $\sum_{i=1}^{d} w_i = 1$ If the short position is not allowed then $w_i \geq 0$ for all $t = 1, \cdots, d$. Let $p(\mathbf{r})$ be the joint distribution of the uncertain return of the assets, the probability of r_p exceeding a certain amount r^* is given by

$$\int_{r_p > r^*} p(\mathbf{r}) d\mathbf{r} \quad \text{then is equivalent to} \quad \Psi(\mathbf{w}, \xi) = \int_{f(w,r) \leq \xi} p(\mathbf{r}) d\mathbf{r} \tag{5}$$

where $\Psi(\mathbf{w}, \xi)$ represents the cumulative distribution function for the associated loss w. Assuming $\Psi(\mathbf{w}, \xi)$ is continuous with respect to ξ, $\text{VaR}(\alpha)$ and $\text{CVaR}(\alpha)$ for the loss $f(\mathbf{w}, \mathbf{r})$ associated with $\mathbf{w}$ any probability level $\alpha \in (0, 1)$ can be defined by

$$\text{VaR}(\alpha, \mathbf{w}) = \min\{\xi \in \mathbb{R} : \Psi(\mathbf{w}, \xi) \geq \alpha\} \tag{6}$$

$$\text{CVaR}(\alpha, \mathbf{w}) = \frac{1}{1-\alpha} \int_{f(\mathbf{w},\mathbf{r}) \geq \text{VaR}(\alpha,\mathbf{w})} f(\mathbf{w}, \mathbf{r}) p(\mathbf{r}) \, d\mathbf{r} \tag{7}$$

Equation (6) is read as the smallest value ξ such that the probability $P[f(\mathbf{w}, \mathbf{r}) > \xi]$ of a loss exceeding ξ is not larger that $1-\alpha$. Therefore, VaR_α presents $(1-\alpha)$-quantile of the loss distribution $\Psi(\mathbf{w}, \xi)$ and CVaR_α presents the conditional expected loss associated with $\mathbf{w}$ if VaR_α is exceeded. Following [19], the $\text{CVaR}(\alpha)$ of the loss associated with any $\mathbf{w}$, it is found

$$\text{CVaR}(\alpha) = \min_{\xi \in \mathbb{R}} F_\alpha(\mathbf{w}, \xi), \tag{8}$$

with

$$F_\alpha(\mathbf{w}, \xi) = \xi + \frac{1}{1-\alpha)} \int_{\mathbf{r} \in \mathbb{R}^n} \max[f(\mathbf{w}, \mathbf{r}) - \xi, 0] \, p(\mathbf{r}) \, d\mathbf{r} \tag{9}$$

Now, the conditional value-at-risk, CVaR, is defined as the solution of an optimization problem

$$\text{CVaR}(\alpha, \mathbf{w}) = \xi + \frac{1}{1-\alpha} \int_{\mathbf{r}\in\mathbb{R}^n} \max\{(f(\mathbf{w}, \mathbf{r}) - \xi)\}\ p(\mathbf{r})\ d\mathbf{r} \tag{10}$$

where α is the probability level such that $0 < \alpha < 1$. CVaR also is known as Mean Excess Loss, Mean Shortfall (Expected Shortfall), or Tail Value-at-Risk, see [1]. However, for general distributions, including discrete distributions, CVaR is defined as the weighted average of VaR and losses strictly exceeding VaR, see [18]. Some properties of CVaR and VaR and their relations are studied in [1] and [17]. For general distributions, CVaR, which is a quite similar to VaR measure of risk has more attractive properties than VaR. CVaR is sub-additive and convex see [18]. Moreover, CVaR is a coherent measure of risk, proved first in [17], see also [1] and [18].

Therefore, the problem of minimizing the Conditional Value at Risk can thus be formulated as the following:

$$\text{minimize} \quad \xi + \frac{1}{1-\alpha} \int_{\mathbf{r}\in\mathbb{R}^n} \max\{(f(\mathbf{w}, \mathbf{r}) - \xi)\}\ p(\mathbf{r})\ d\mathbf{r} \tag{11}$$

$$\text{subject to} \quad \sum_{i=1}^{n} w_i = 1, \quad x_i \geq 0 \tag{12}$$

$$-\mathbf{w}^T E(\mathbf{r}) \leq -r^* \tag{13}$$

In this case the feasibility set is $\mathcal{X}$ defined on region satisfying (12) and (13). Set $\mathcal{X}$ is convex (it is polyhedral, due to linearity in constraint (12) and (13)). The optimization problem (11) -(13) are a convex programming. This problem is solved using *PortfolioCVaR class* of MATLAB R2013a.

4 Empirical studies

In this section, the models of stock portfolio risk measurement and management (described in the previous sections) are implemented to a hypothetical portfolio composed by 4 Indonesian Blue Chip stocks: ASSI=Astra International Tbk, BMRI=Bank Mandiri (Persero) Tbk., PTBA=Tambang Batubara Bukit Asam Tbk., INTP=Indocement Tunggal Perkasa Tbk. The historical data are recorded during the period of 6 November 2006 to 2 August 2013. Precisely, the statistics of the 4 stocks are described in Table 1.

Table 1 reflects the mean, standard deviation, skewness, and kurtosis of the daily returns of four stocks over the period from 6 November 2006 to 2 August 2013. The kurtosis of all stock returns exceeds the kurtosis of normal distribution (3.0) substantially. The Jarque-Bera tests of the null hypothesis that the return

Table 1. Descriptive Statistics

Statistics	ASII	BMRI	PTBA	INTP
Mean	0.0009	0.0006	0.0006	0.0009
Standard deviation	0.0276	0.0271	0.0317	0.0307
Skewness	0.1306	0.3690	-0.2404	0.370
Kurtosis	10.0123	8.2807	13.0502	45.0897
Jarque-Bera Test (5%) 1 = reject H_0	1	1	1	1
Jarque-Bera Statistics	35700	20612	73398	12848
Jarque-Bera p-Values	0.001	0.001	0.001	0.001
Jarque-Bera Crit-Value	5.9586	5.9586	5.9586	5.9586

distribution follow a normal distribution against the alternative that the return do not come from a normal distribution. As seen from Table 1. that the test statistics exceed the critical value at the 5% level of significant, this means that all stock returns are not normally distributed, it shows a fat tailed distribution. This means that the probability of extreme events is higher than the probability of extreme events under the normal distribution. Therefore, capturing the stock returns using normal distribution could be underestimated. When the skewness of the return data are considered, it shows that all returns are right skewed except for PTBA. This suggests that the returns are not symmetry. Again, capturing by normal distribution may result in misleading conclusions.

Figure 1 shows the stock prices presentation in the form of relative price movements. The initial level of each stock has been normalized to unity to see how their performance compared relatively to each other over the historical period of 6-11-2006 to 2-08-2013. It is seen that PTBA dominates the price to three other prices.

Figure2 shows the normal distribution presented by the dashed straight lines and the pareto tail distribution is presented by the curved dash lines. The figure reflect the comparison between the fit produced by the pareto tail distribution to the fit produced by the normal distribution. By producing the piecewise distribution object, one can interpolate each tail of the distribution within the interior of the CDF and extrapolate to estimate quantiles outside the historical record. By interpreting the figure, it is concluded that the normal distribution are far from pareto distribution.

After carrying out extrapolation to the Pareto tails and interpolation to the smoothed interior, the copula parameters then are estimated by simulating jointly-dependent uniform distribution using the function copularnd. The results obtained are then transformed into daily centered returns via the inverse CDF of each index. The results are shown in Figure 3. These simulated centered returns are consistent with those obtained from the historical dataset.

Table 2 reflects the value of VaR and CVaR simulated by Gaussian, t copula, and multivariate normal at 1% and 5% significant levels. The multivariate nor-

In this discussion, historical returns as a scenario input are used to the model, without making any assumptions about the distribution of the scenario variables. Var and CVaR produced from MV are compared with VaR and CVaR produced by copula simulation, the output can be seen in Table 2. It is seen that the VaR and CVaR are not significantly different simulated either by Gaussian or t copula. This is due to the fact that Gaussian and t copula come from the same family, that is elliptic copula. When the degree of freedom of t copula are getting bigger, t copula will converge to Gaussian copula.

Figure4 shows the CVaR-efficient frontier of a portfolio in term of expected returns and CVaR which is scaled for the risk confidence level $\alpha = 0.99$. The discontinuous lines represent the efficient frontier curve simulated using Gaussian copula, while the continuous lines represent the efficient frontier curve simulated using t copula. Note, that for a given CVaR, simulated by t copula portfolio has a higher expected return than that simulated by Gaussian copula. However, the difference between the Gaussian and the t copula approach in simulating CVaR is not quite significant (see also Table 2). This is due to the fact that Gaussian copula and t copula come from the same family. However, this agreement between the two copulas should not be misleading in deciding that copula from the same family will produce the same result. The obtained results depend on the characteristics of dataset.

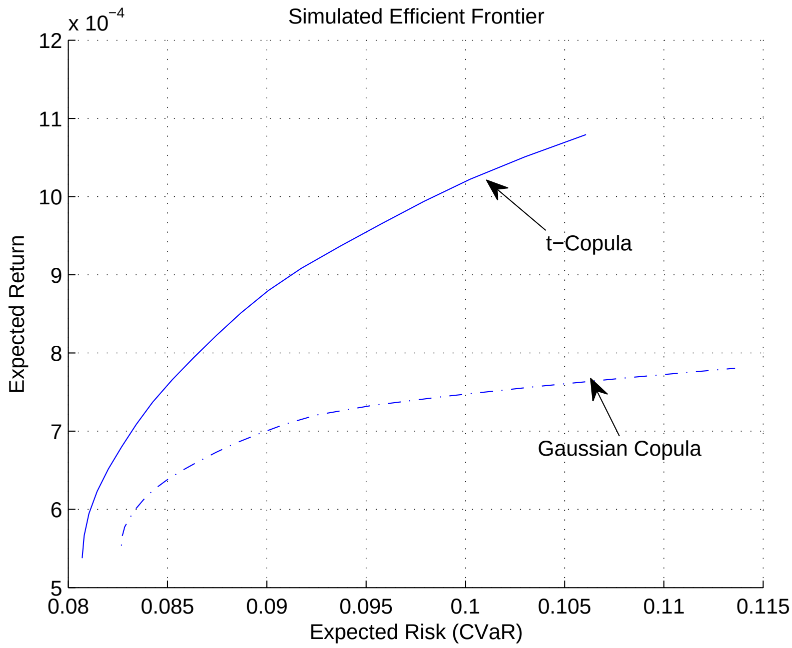

Fig. 4. Efficient frontier simulated by Gaussian and t copula

5 Concluding remark

The paper discuss the estimation and optimization of CVaR as an alternative to VaR using elliptical copula simulations. This paper suggests a measurement of CVaR using elliptic and archimedean copula simulations. This is done by simulating pairwise bivariate copula. Once pairwise correlations of the simulated returns are calculated the CVaR is optimized. The case study showed that the optimization algorithm, which is based on PortfolioCVaR class of MATLAB R2013a is very stable and efficient. The approach can handle large number of instruments and scenarios and can easily be extended to other copula families.

References

1. Acerbi, C., Nordio, C., Sirtori, C. *Expected Shortfall as a Tool for Financial Risk Management.* Working Paper, can be downloaded from: http://gloria-mundi.com/Library_Journal.asp
2. Artzner, P., Delbaen F., Eber, J. M. and D. Heath (1999), *Coherent Measures of Risk.* Mathematical Finance, 9, 203-228.
3. Breymann, W., Dias, A. and Embrechts, P. (2003). *Dependence structures for multivariate high-frequency data in finance.* Quant. Finance 1: 1-14.
4. Brooks, C., and Kat, H. M. (2002). *The statistical properties of hedge fund index returns and their implications for investors.* Journal of Alternative Investments 5, 2644.
5. Chen, X. and Fan, Y. (2006). *Estimation and model selection of semiparametric copula-based multivariate dynamic models under copula misspesification.* J. Econometrics 135:125-154.
6. Cherubini U., S. Mulinacci, F. Gobbi, and S Romagnoli. (2012). Dynamic Copula Methods in Finance. Wiley Finance Series
7. Cherubini, U., Luciano, E. and Vecchiato, W. (2004). *Copula Methods in Finance.* John Wiley & Sons, New York.
8. Clemente A. D. and C. Romano. (2004). *Measuring and Optimizing Portfolio Credit Risk: A Copula-based Approach.* Economic Notes by Banca Monte dei Paschi di Siena Sp. A. vol. 33, no. 3, pp. 325357
9. Embrechts P., Kluppelberg C. and Mikosch T. (1997). *Modelling Extremal Events for Insurance and Finance.* Berlin: Springer.
10. Hotta, L.K., Lucas, E.C., Palaro, H.P. (2008). *Estimation of VaR using copula and extreme value theory. Multinational Finance Journal.* Vol. 12, Issue 3/4, p.205-221.
11. HuangJ.J et.al. (2009). *Estimating Value at Risk of Portfolio by Conditional Copula-GARCH method. Insurance*: Mathematics and Economics. Doi:10.1016
12. MATLAB version 8.01.0 (R2013a). (2013) Natick, Massachusetts: The MathWorks Inc.
13. Nikoloulopoulos, A. K. and Karlis, D. (2009). Finite normal mixture copulas for multivariate discrete data modeling. Journal of Statistical Planning and Inference, 139:3878-3890.
14. Nikoloulopoulos, A. K., Joe, H., and Li, H. (2012). Vine copulas with asymmetric tail dependence and applications to financial return data. Computational Statistics & Data Analysis, 56:3659-3673.
15. Nelsen R. B. (2006). *An Introduction to Copulas.* Second Editions. Springer Science+Business Media, Inc.
16. Novak S. Y. (2012). Extreme Value Methods with Applications to Finance. A Chapman and Hall Book.
17. Pflug G. Ch. (2000). *Some Remarks on the Val-ue-At-Risk and the Conditional Value-At-Risk* In: S. Uryasev, Ed., Probabilistic Constrained Optimization: Methodology and Applications, Kluwer Academic Publishers.
18. Rockafellar R. T. and S. Uryasev. (2000). *Optimization of Conditional Value-at-Risk.* Journal of Risk, Vol. 2, pp. 21-41
19. Rockafellar, R. T. and S. Uryasev. (2002). *Conditional Value-at-Risk for General Loss Distributions*, Journal of Banking and Finance, Vol. 26, pp. 1443-1471.